Ultrasonic Phased Arrays and Interactive Reflectivity Tomography

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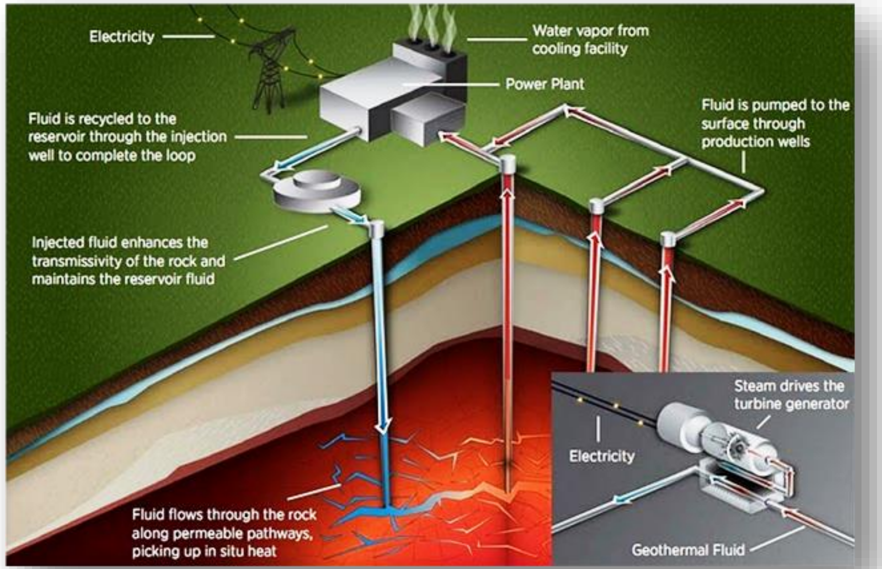
Motivation

Nuclear Power Plants

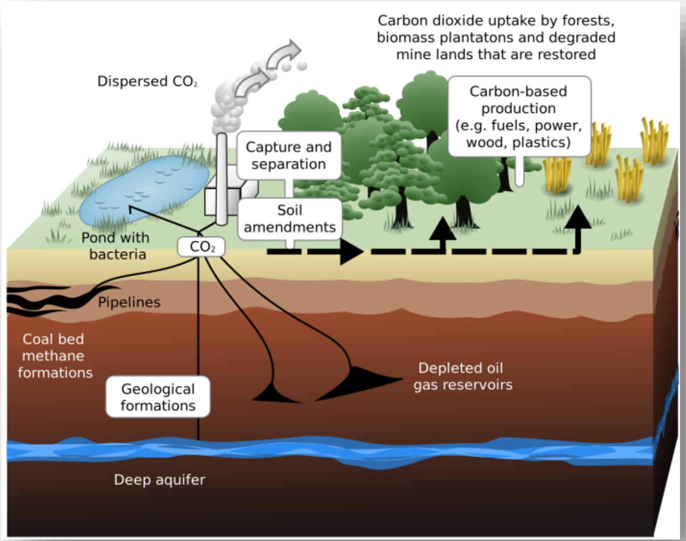
- Containment Buildings
- Reactor buildings
- Fuel pools
- Cooling towers

Oil and Gas

Geothermal

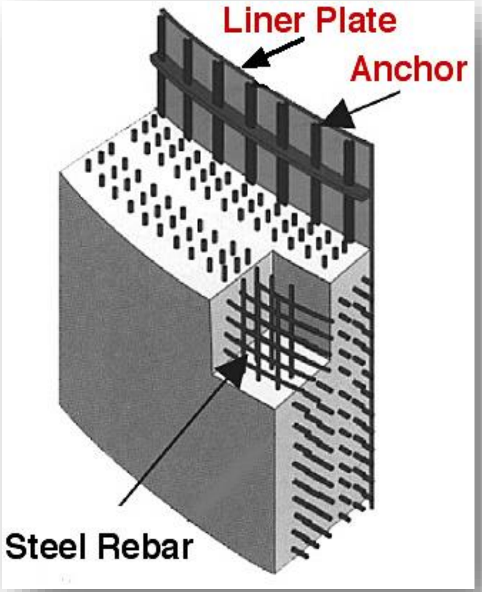


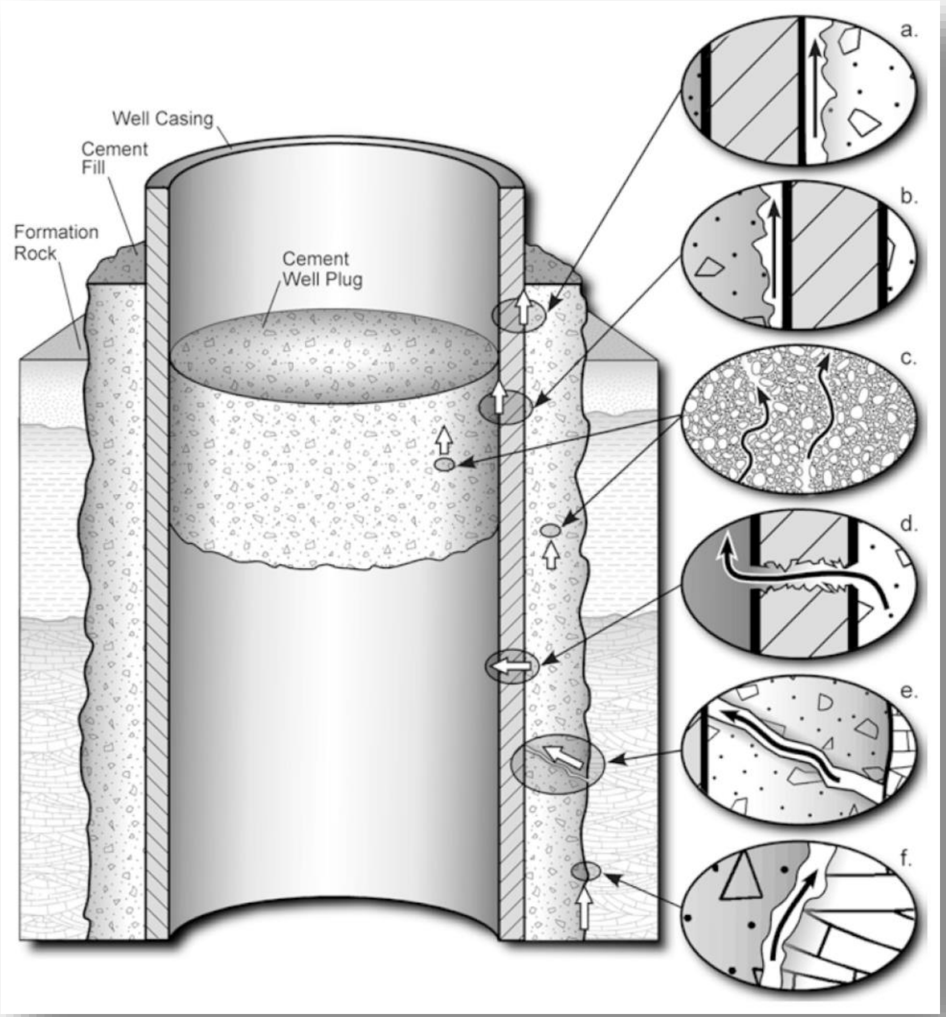
CCS



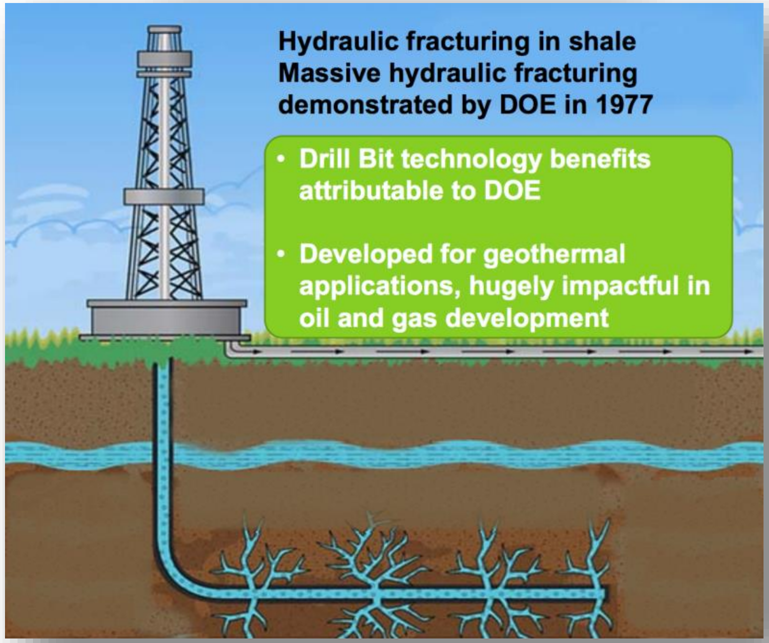
Motivation: Needs

- Low density matrix
- Mixture of cement, aggregate, rocks, and water
- High density reinforcement

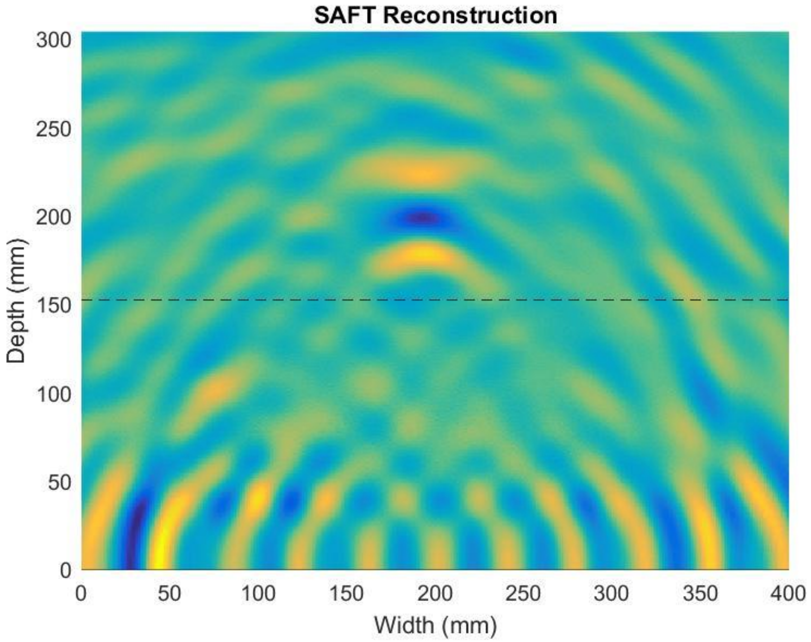




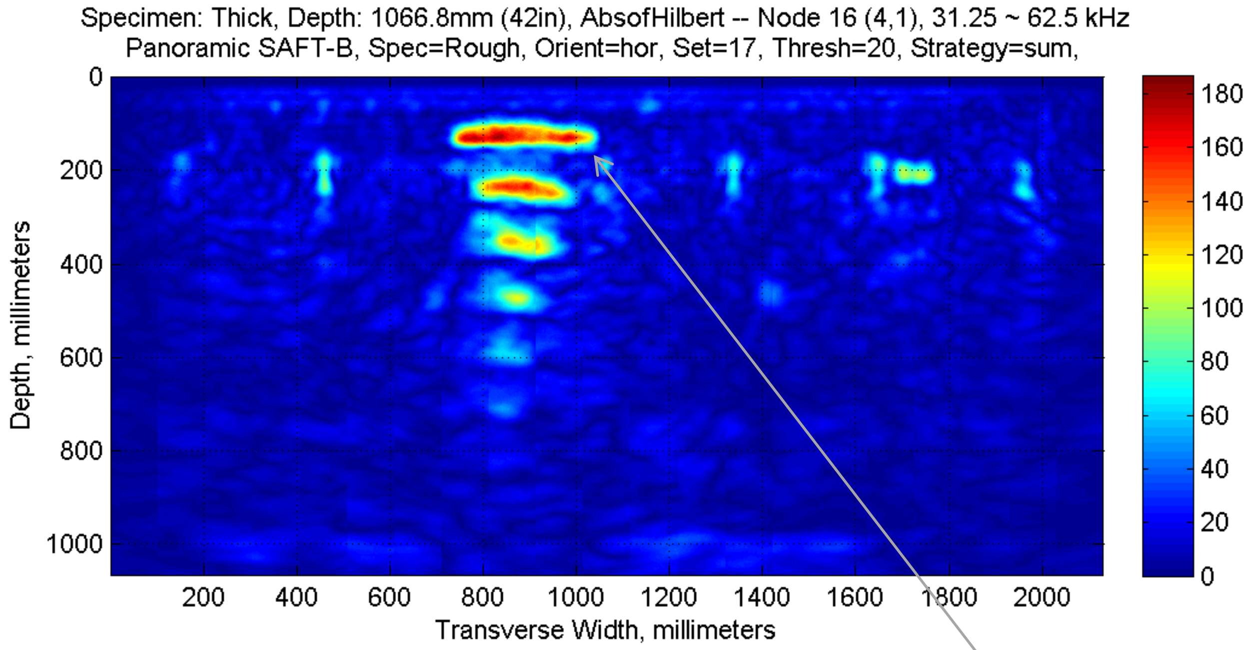
[Gasda 2004]



Motivation: Synthetic Aperture Focusing Technique (SAFT)



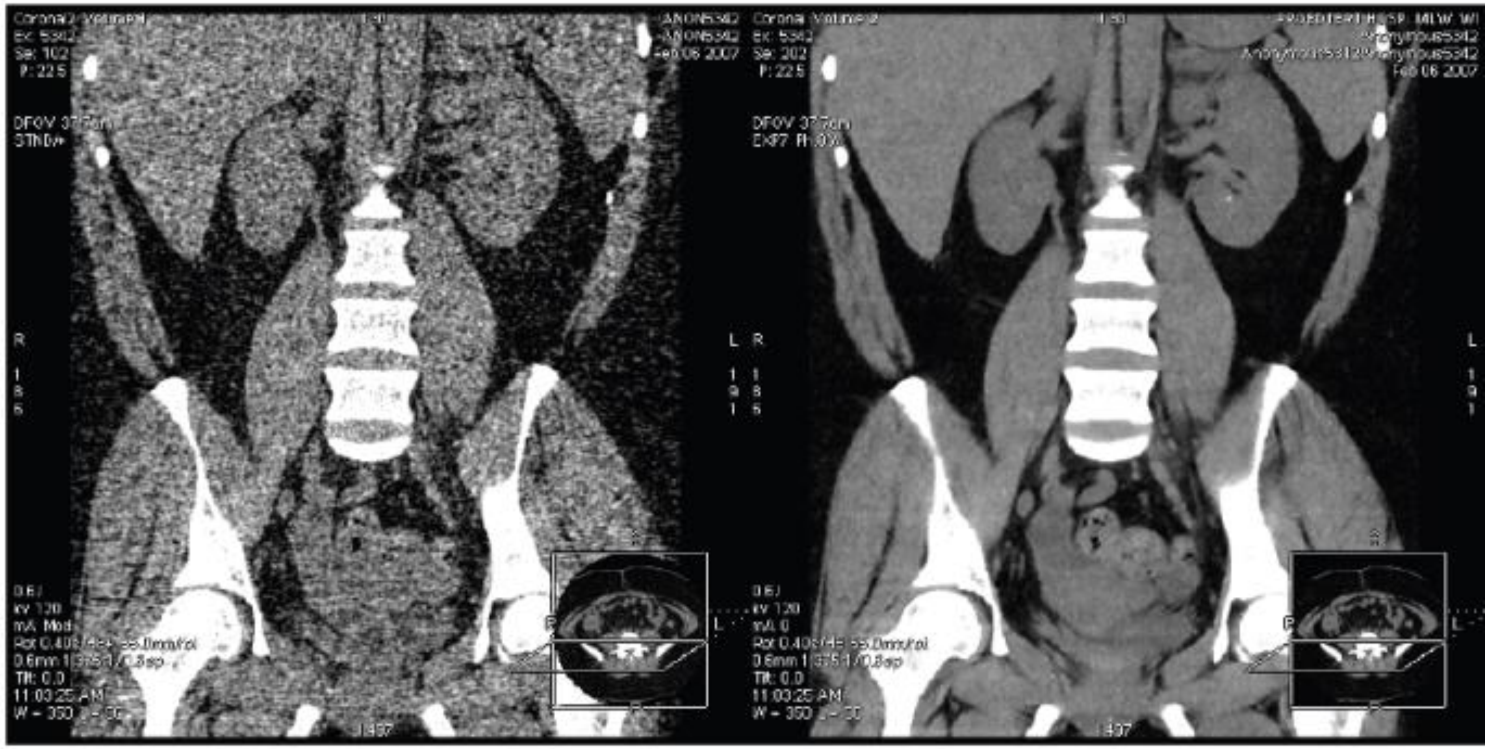
Homogenous medium



Non-homogenous
Thick Concrete Specimen

Plexiglas

Motivation: Model-Based Iterative Reconstruction (MBIR)



State-of-the-art 3D Recon GE MBIR

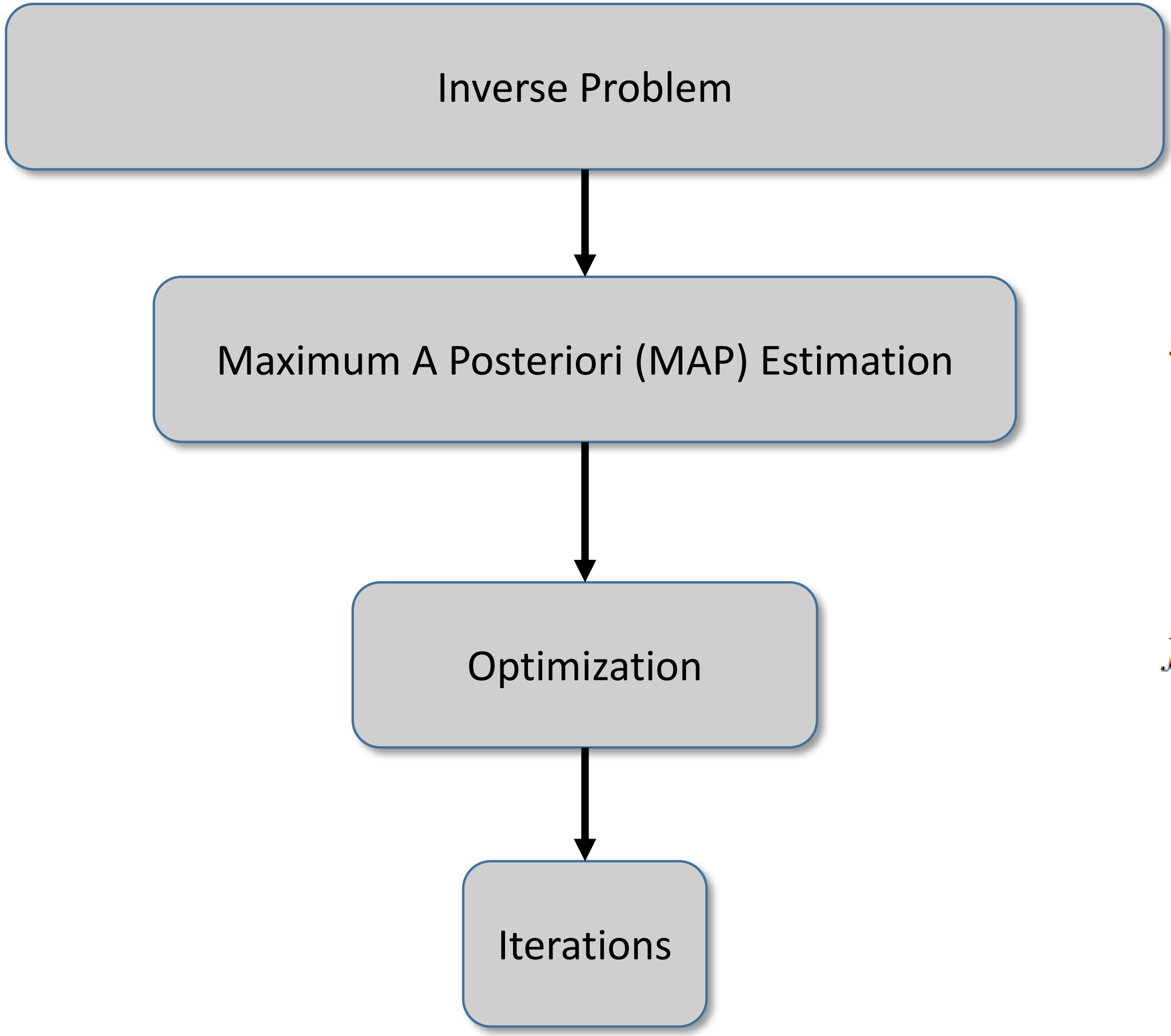
MBIR looks for the best solution that fits the data

*"Rather than making the **"purest"** measurement, make the most **informative** measurement."*
--Charlie Bouman, Professor Purdue University

Goal

Implement MBIR to ultrasonic system for thick non-homogeneous concrete structure.

Background On MBIR



$$Y = AX + W$$

$$\hat{x}_{MAP} = \arg\min_{x \in \Omega} \left\{ -\log p_{y|x}(y|x) - \log p_x(x) \right\}$$

$$f(x) = \frac{1}{2} ||y - Ax||_{\Lambda}^{2} + \sum_{\{s,r\} \in \mathcal{P}} b_{s,r} \rho(x_s - x_r)$$

Background On MBIR: MAP Estimation

$$\hat{x}_{MAP} = \arg\min_{x\in\Omega} \{\boxed{-\log p_{y|x}(y|x)}\boxed{-\log p_x(x)}\}$$

Probabilistic model of the measurements (Forward Model)

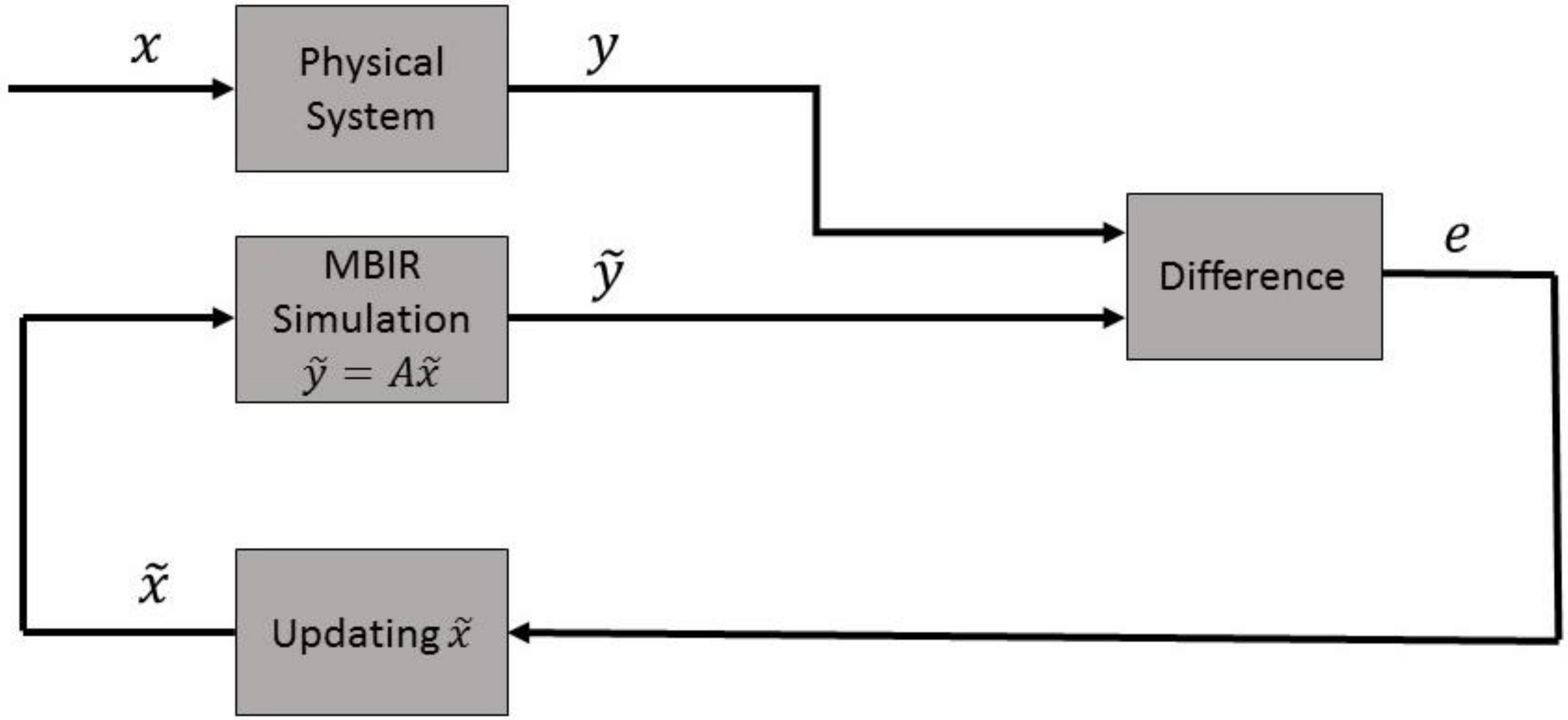
Probabilistic model of the image (Prior Model)

$$f(x) = \boxed{\frac{1}{2}||y - Ax||^2_\Lambda} + \boxed{\sum_{\{s,r\}\in\mathcal{P}} b_{s,r}\rho(x_s - x_r)}$$

For $W \sim N(0, \Lambda^{-1})$

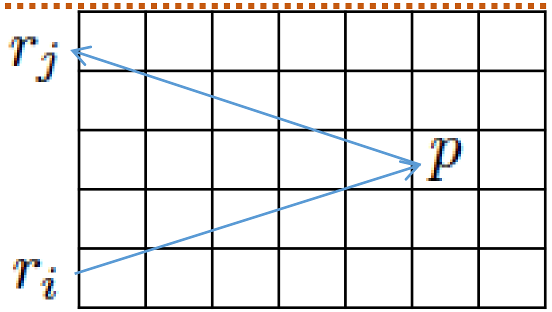
For X ~ pair-wise Gibbs Distribution

Background On MBIR (cont.)



Ultrasonic MBIR: Forward Model

- Homogenous, isotropic, and linear medium



Source Intensity

$$S(f) = \mathscr{F}\left\{s(t)\right\}$$

Intensity Reflectivity Coefficient (IRC)

$$x(p)$$

- Propagation model

$$G(r_i, p, f) = \tau_1 \tau_2 \exp\left\{-(\alpha(f) + j\beta(f))||p - r_i||\right\}$$

  - $\alpha(f) \approx \alpha_0 |f|$ is the attenuation coefficient
  - $\beta(f)) \approx \frac{2\pi f}{c}$ is the phase delay

- Measurements

$$\begin{aligned} Y_{i,j}(p, f) &= S(f) G(r_i, p, f) x(p) G(p, r_j, f) \\ &= \tau_1^2 \tau_2^2 x(p) S(f) \exp\left\{-\left(\alpha_0 c |f| + j 2\pi f\right) T_{i,j}(p)\right\} \end{aligned}$$

$$y_{i,j}(p, t) = h(T_{i,j}(p), t - T_{i,j}(p))\, x(p)$$

Ultrasonic MBIR: Forward Model (cont.)

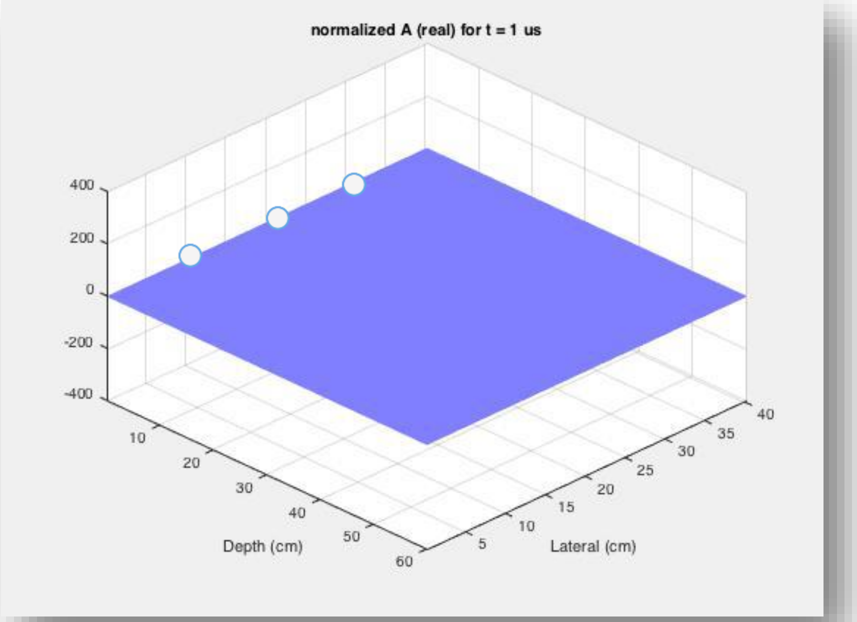
$$T_{i,j}(p) = \frac{||p - r_i|| + ||p - r_j||}{c}$$

$$A_{i,j}(T_{i,j}(p), t) = h(T_{i,j}(p), t - T_{i,j}(p))$$

$$y_{i,j}(t) = \int_{\mathbb{R}^3} y_{i,j}(p,t)dp = \boxed{\int_{\mathbb{R}^3} A_{i,j}(T_{i,j}(p),t)x(p)dp}$$

$$\begin{bmatrix} a_{1111} & a_{1112} & \cdots & a_{111N} \\ a_{1121} & a_{1122} & \cdots & a_{112N} \\ \cdots & \cdots & \cdots & \cdots \\ a_{11N_t1} & a_{11N_t2} & \cdots & a_{11N_tN} \\ a_{ij11} & a_{ij12} & \cdots & a_{ij1N} \\ a_{ij21} & a_{ij22} & \cdots & a_{ij2N} \\ \cdots & \cdots & \cdots & \cdots \\ a_{ijN_t1} & a_{ijN_t2} & \cdots & a_{ijN_tN} \\ \cdots & \cdots & \cdots & \cdots \\ a_{N_TN_TN_t1} & a_{N_TN_TN_t2} & \cdots & a_{N_TN_TN_tN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} y_{111} \\ y_{112} \\ \vdots \\ y_{11N_t} \\ y_{ij1} \\ y_{ij2} \\ \vdots \\ y_{ijN_t} \\ \vdots \\ y_{N_TN_TN_t} \end{bmatrix}$$

$$A \qquad X = Y$$



Ultrasonic MBIR: Prior Model

$$-\log p_x(x) = \sum_{\{s,r\}\in\mathcal{P}} b_{s,r}\rho(x_s - x_r)$$

X ~ pair-wise Gibbs Distribution

- $\mathcal{P}$ is the set of all unordered neighboring pixel pairs $\{s, r\}$ such that $r \in \partial s$.
- Q-Generalized Gaussian Markov Random Field (QGGMRF)
- It has continuous first and second derivative near Δ = 0.

$$\rho(\Delta) = \frac{|\Delta|^p}{p}\left(\frac{\left|\frac{\Delta}{T}\right|^{q-p}}{1+\left|\frac{\Delta}{T}\right|^{q-p}}\right)$$

  - $1 \leq p < q$ for guaranteed function convexity
  - Usually, *p* = 2 and q is close to 1.
    - $|\Delta| < T$ preserves details in low contrast regions
    - $|\Delta| > T$ preserves edges

Ultrasonic MBIR: Optimization

$$\hat{x}_{MAP} = \arg\min_{x\in\Omega} \left\{ \frac{1}{2}||y - Ax||^2_\Lambda + \sum_{\{s,r\}\in\mathcal{P}} b_{s,r}\rho(x_s - x_r) \right\}$$

- Iterative Coordinate Descent (ICD) algorithm
  - Fast and stable algorithm
  - Better for defining high frequency components (i.e., edges)
  - Usually initialize object estimate with lower resolution reconstruction
    - E.g., back-propagation of measured signals $A^{-1}y$

ICD Algorithm using Majorization of Prior:

Initialize $e \leftarrow y - Ax$
For K iterations {
  For each pixel $s \in S$ {

$$\tilde{b}_{s,r} \leftarrow \frac{b_{s,r}\rho'(x_s - x_r)}{2(x_s - x_r)}$$

$$\theta_1 \leftarrow -e^t \Lambda A_{*,s} + \sum_{r\in\partial s} 2\tilde{b}_{s,r}\,(x_s - x_r)$$

$$\theta_2 \leftarrow A^t_{*,s}\Lambda A_{*,s} + \sum_{r\in\partial s} 2\tilde{b}_{s,r}$$

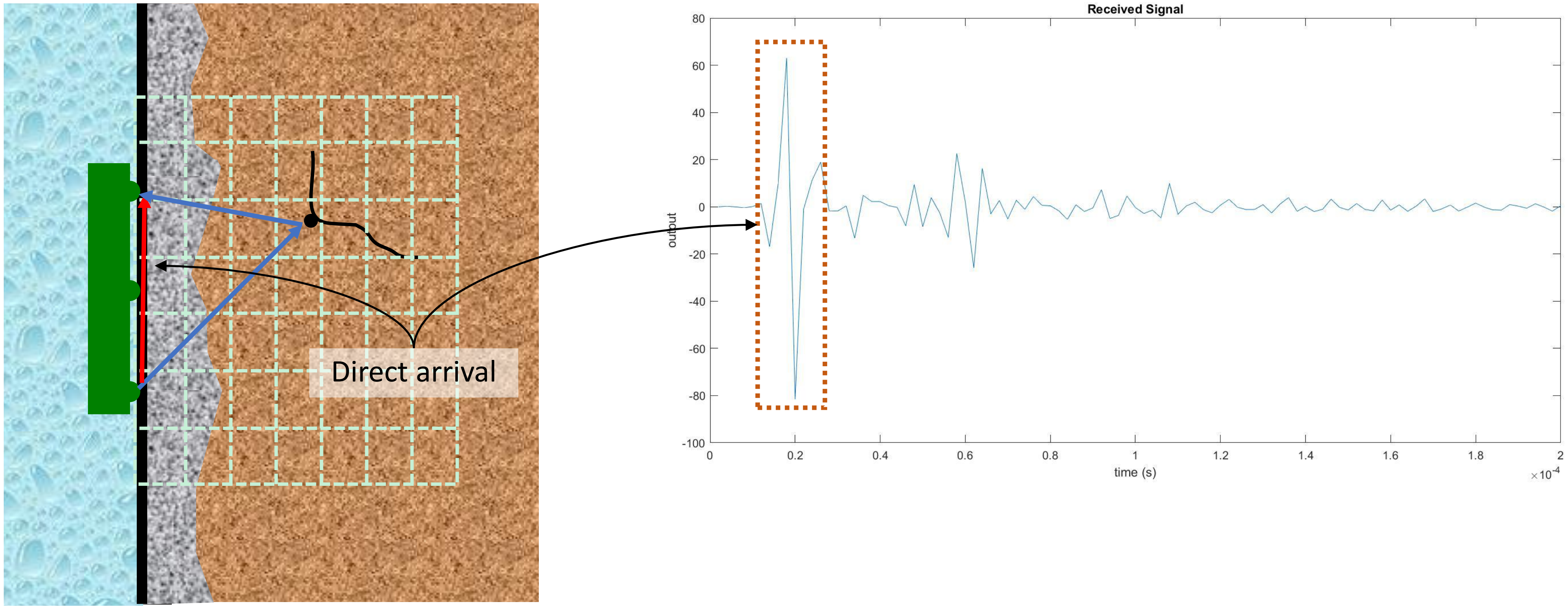
$$\alpha^* \leftarrow \text{clip}\left\{\frac{-\theta_1}{\theta_2}, [-x_s, \infty)\right\}$$

$$x_s \leftarrow x_s + \alpha^*$$

$$e \leftarrow e - A_{*,s}\alpha^*$$

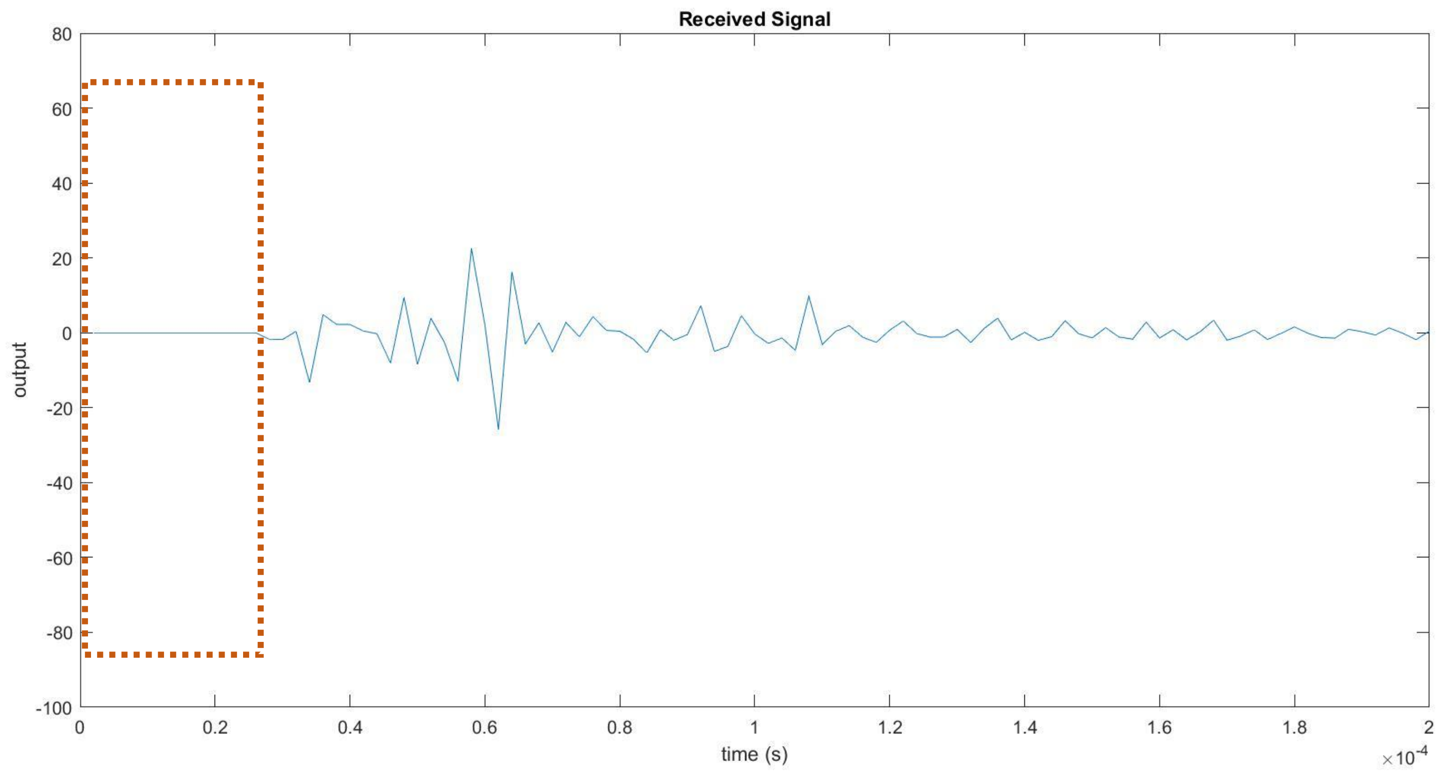
  }
}

Forward Model Upgrade: Direct Arrival Signal



Forward Model Upgrade: Direct Arrival Signal

Old solution:



Forward Model Upgrade: Direct Arrival Signal

New solution:

Replace Forward Model and MAP Estimate

$$y_{i,j}(t) = -\int_{\mathbb{R}^3} A_{i,j}(T_{i,j}(p), t)x(p)dp \qquad T_{i,j}(p) = \frac{||p - r_i|| + ||p - r_j||}{c}$$

$$(\hat{X})_{MAP} = \underset{(X)}{\text{argmin}} \left\{ \frac{1}{2}||y - AX||_\Lambda + u(X) \right\}$$

With

$$y_{i,j}(t) = -\int_{\mathbb{R}^3} A_{i,j}(T_{i,j}(p), t)x(p)dp + \boxed{A_{i,j}(T_{i,j}, t)z_{i,j}} \qquad T_{i,j} = \frac{||r_j - r_i||}{c}$$

Direct arrival

$$(\hat{X}, \hat{z})_{MAP} = \underset{(X,z)}{\text{argmin}} \left\{ \frac{1}{2}||y - AX \boxed{- dz}||_\Lambda + u(X) \right\}$$

$$\Longrightarrow \hat{z} = \boxed{(d^t d)^{-1}} d^t(y - AX)$$

$$(d^t d)^{-1} = \begin{bmatrix} \frac{1}{d_1^t d_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{d_2^t d_2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{d_k^t d_k} \end{bmatrix}$$

Forward Model Upgrade: Direct Arrival Signal

New solution:

ICD Algorithm using Majorization of Prior:

Initialize $e \leftarrow y - Ax$
For K iterations {
  For each pixel $s \in S$ {

$$
\begin{aligned}
\tilde{b}_{s,r} &\leftarrow \frac{b_{s,r}\rho'(x_s - x_r)}{2(x_s - x_r)} \\
\theta_1 &\leftarrow -e^t \Lambda A_{*,s} + \sum_{r \in \partial s} 2\tilde{b}_{s,r}(x_s - x_r) \\
\theta_2 &\leftarrow A_{*,s}^t \Lambda A_{*,s} + \sum_{r \in \partial s} 2\tilde{b}_{s,r} \\
\alpha^* &\leftarrow \text{clip}\left\{\frac{-\theta_1}{\theta_2}, [-x_s, \infty)\right\} \\
x_s &\leftarrow x_s + \alpha^* \\
e &\leftarrow e - A_{*,s}\alpha^*
\end{aligned}
$$

  }
}

ICD Algorithm using Majorization of Prior:

Initialize $e \leftarrow y - Ax$
For K iterations {

$$
\boxed{
\begin{aligned}
& z = (d^t d)^{-1} d^t e \\
& e \leftarrow e - dz
\end{aligned}
}
$$

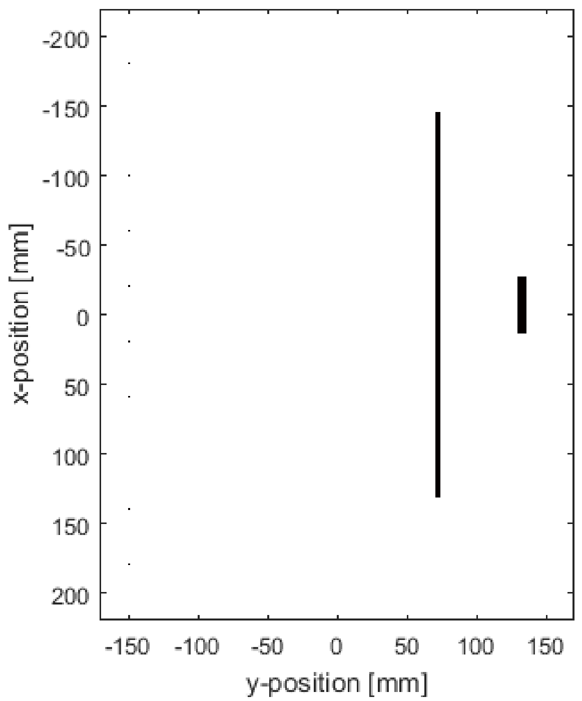
  For each pixel $s \in S$ {

$$
\begin{aligned}
\tilde{b}_{s,r} &\leftarrow \frac{b_{s,r}\rho'(x_s - x_r)}{2(x_s - x_r)} \\
\theta_1 &\leftarrow -e^t \Lambda A_{*,s} + \sum_{r \in \partial s} 2\tilde{b}_{s,r}(x_s - x_r) \\
\theta_2 &\leftarrow A_{*,s}^t \Lambda A_{*,s} + \sum_{r \in \partial s} 2\tilde{b}_{s,r} \\
\alpha^* &\leftarrow \text{clip}\left\{\frac{-\theta_1}{\theta_2}, [-x_s, \infty)\right\} \\
x_s &\leftarrow x_s + \alpha^* \\
e &\leftarrow e - A_{*,s}\alpha^*
\end{aligned}
$$

  }
}

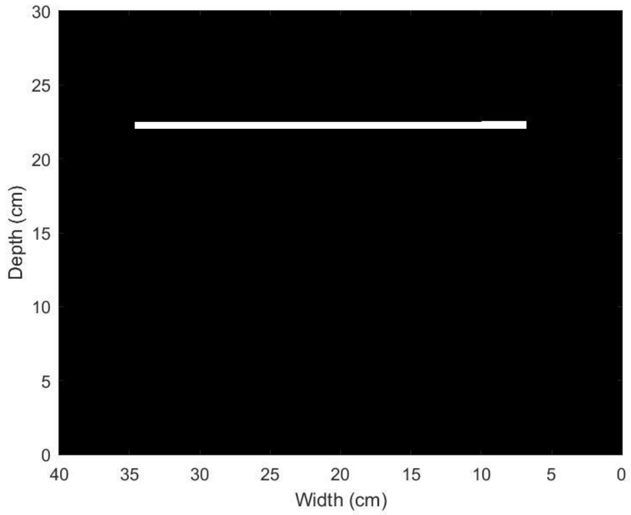
Synthetic Data Reconstruction

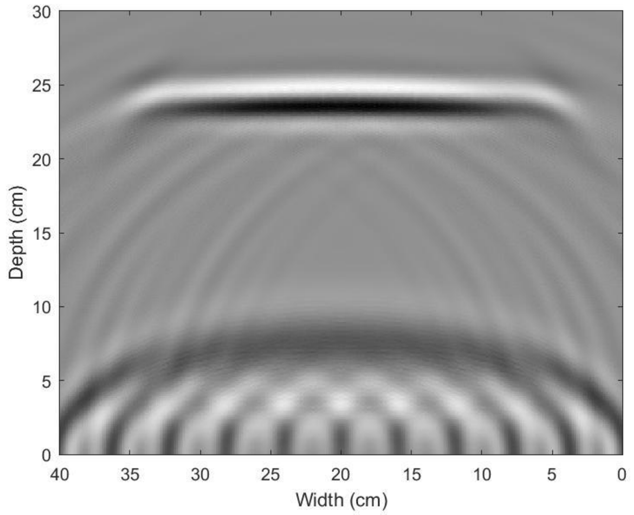
- K-Wave simulation
  - 4 simulations
    - 2 simulations of simple phantoms
    - 2 simulations of corner cases
  - 10-transducer in-line system
  - Input signal: 3-cycle sine wave,52 kHz central frequency
  - Medium size: 40 cm wide, 30 cm thick.

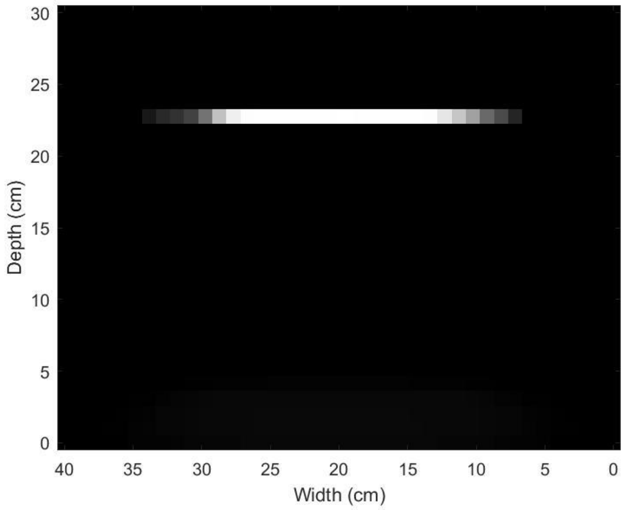


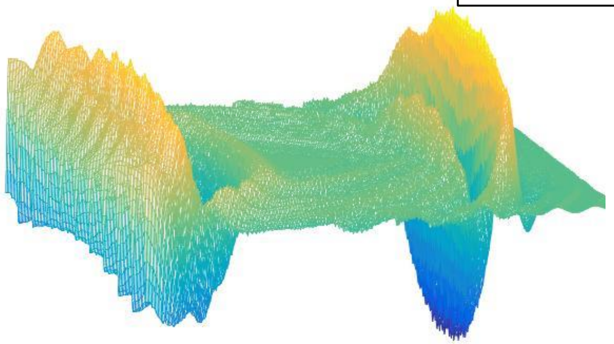
Synthetic Data Reconstruction No. 1

Ground truth | *SAFT* | *MBIR*

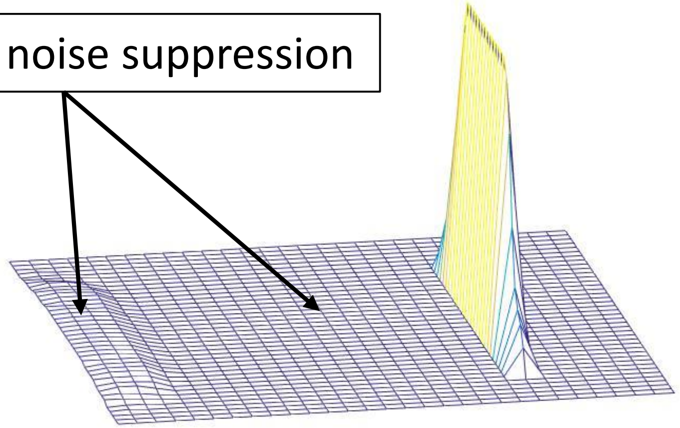








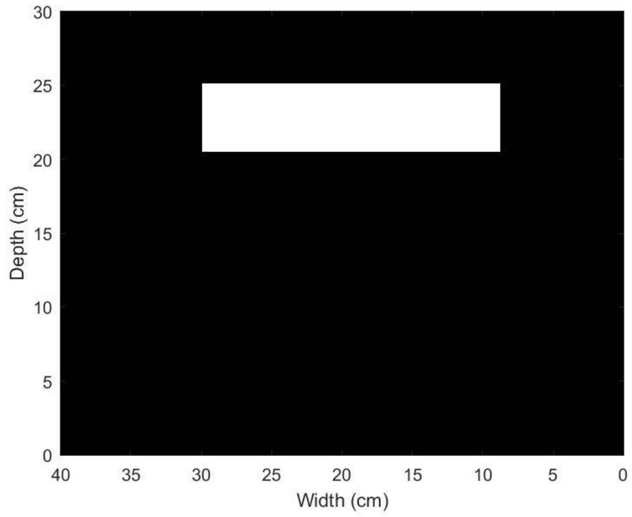
Artifact and noise suppression



MBIR looks for the best solution that fits the data

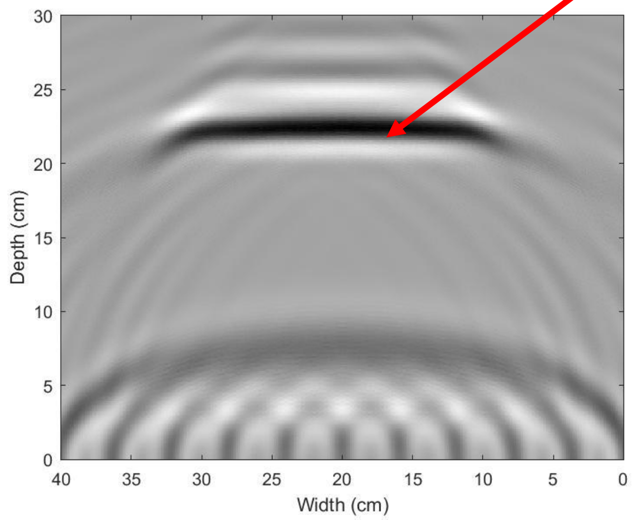
Synthetic Data Reconstruction No. 2

Ground truth

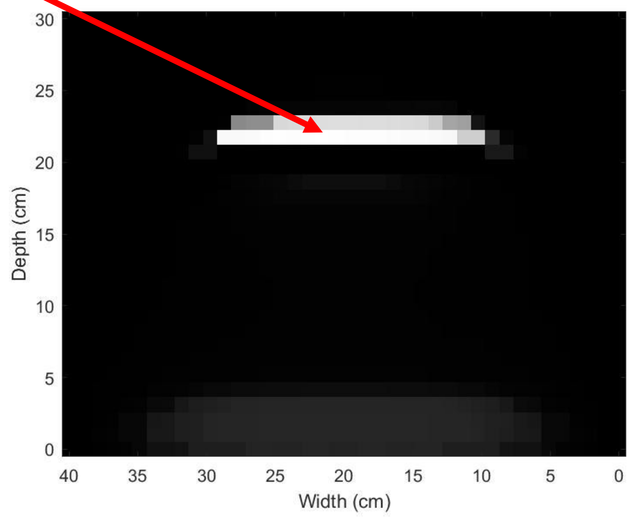


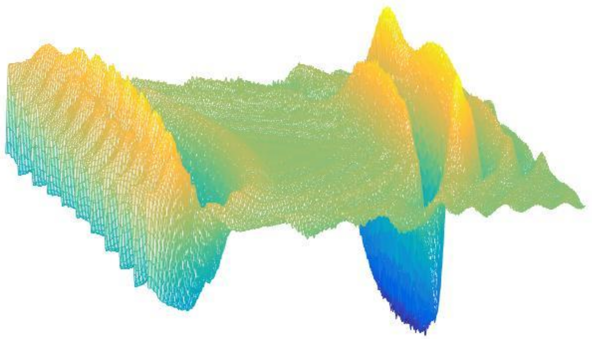
SAFT

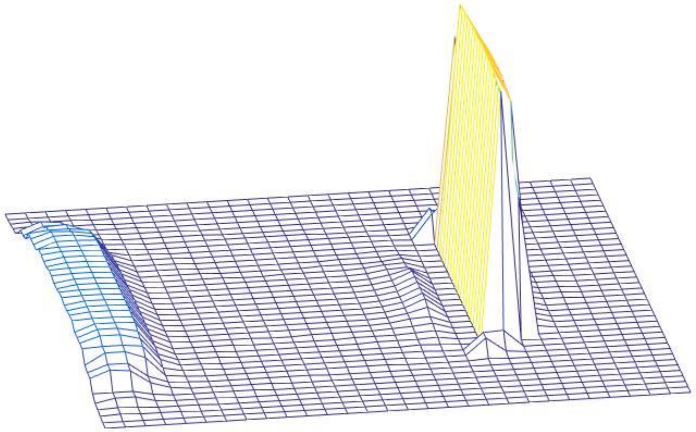
Thin Rectangle



MBIR

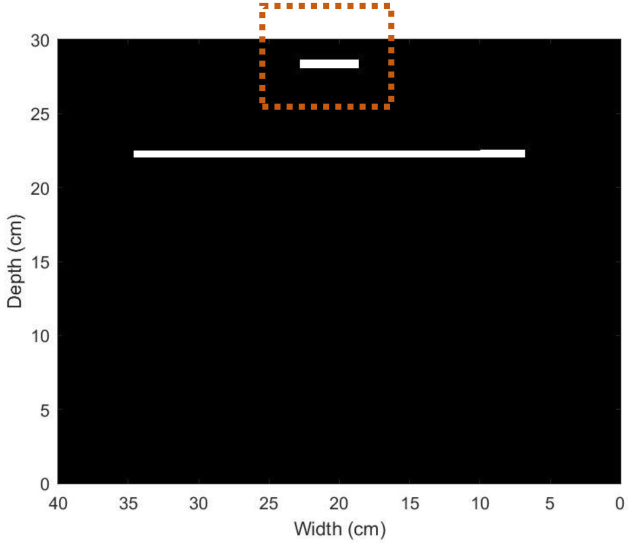




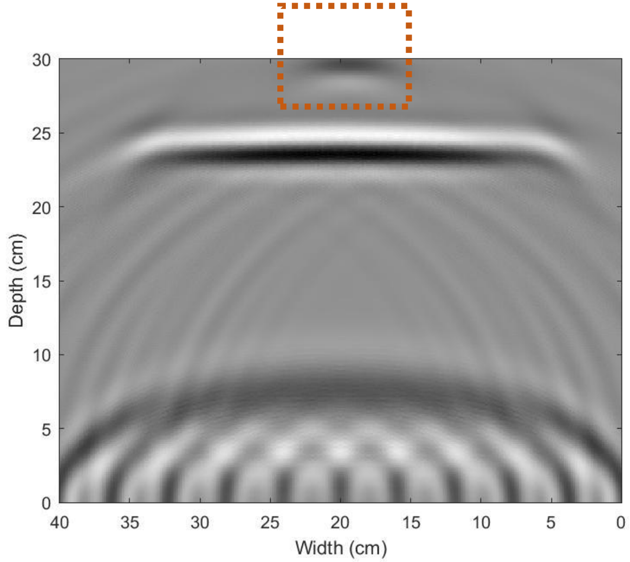


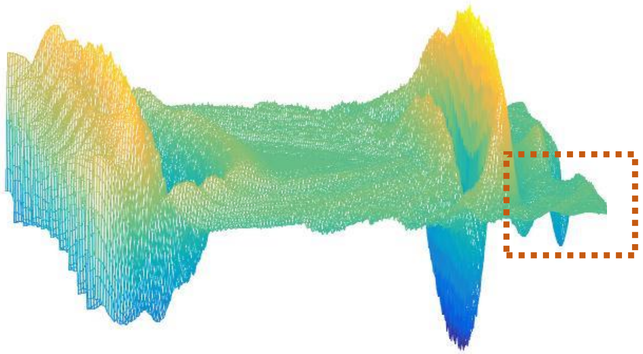
Synthetic Data Reconstruction No. 3

Ground truth

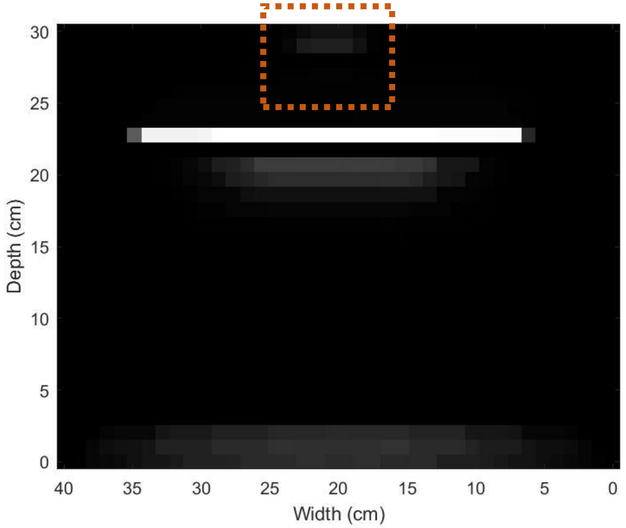


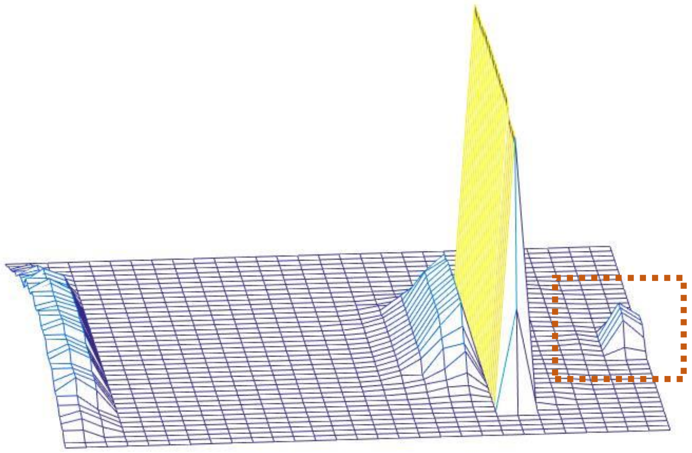
SAFT





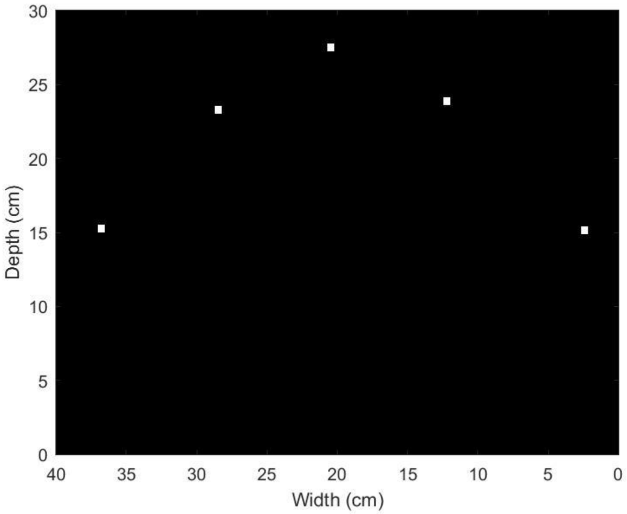
MBIR



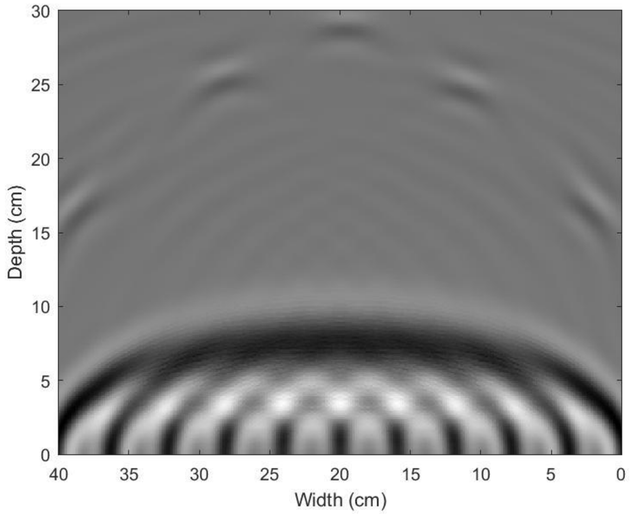


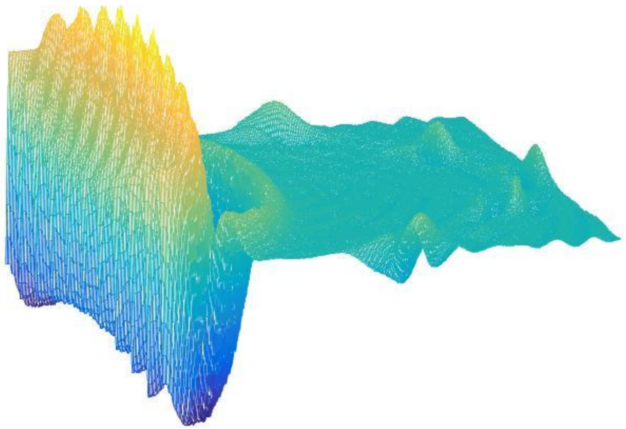
Synthetic Data Reconstruction No. 4

Ground truth

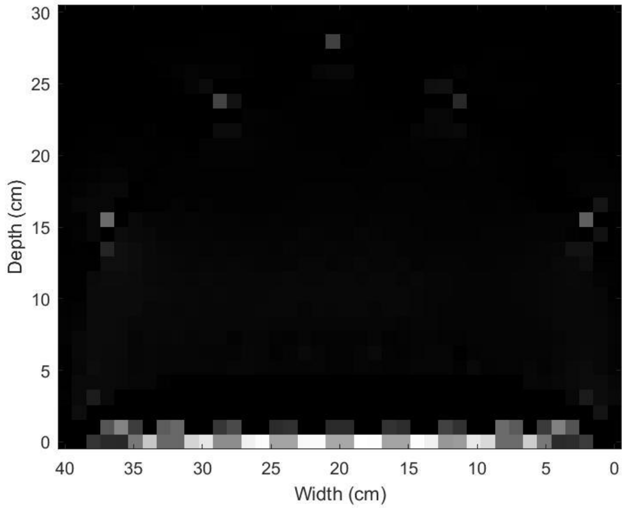


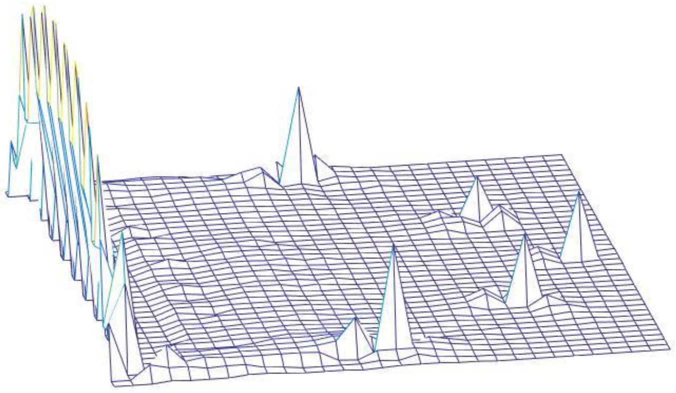
SAFT





MBIR

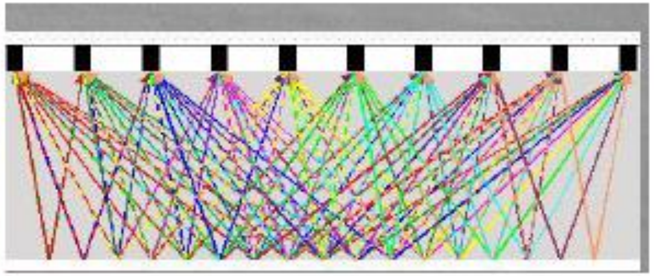




Empirical Data Reconstruction

First Experiment:

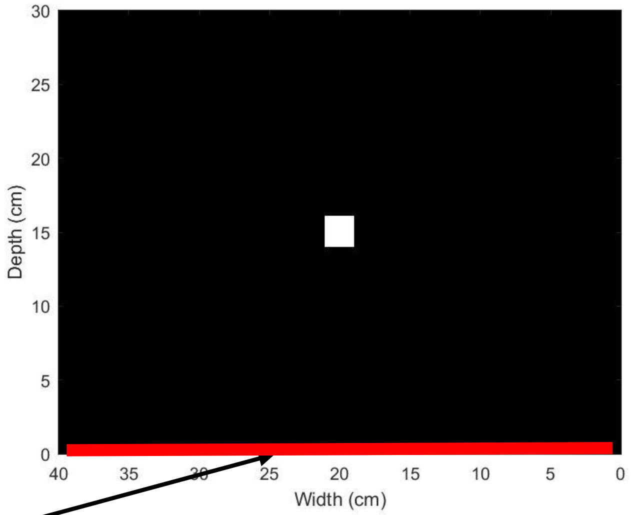
- Cement sample with a rebar inside
  - 10-transducer in-line system
  - Input signal: 52 kHz central frequency
  - Medium size: 40 cm wide, 30 cm thick.

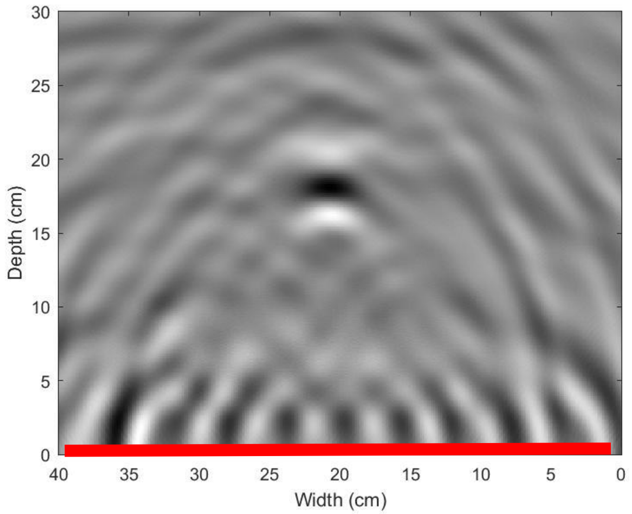

45
Measurement
Pairs

Empirical Data Reconstruction (cont.)

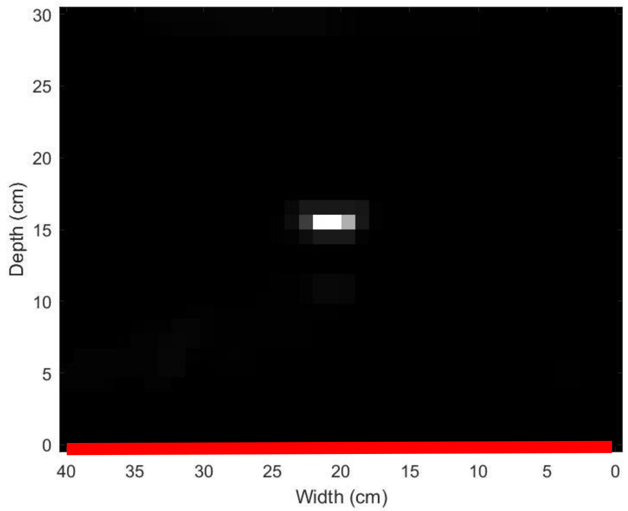
Ground truth



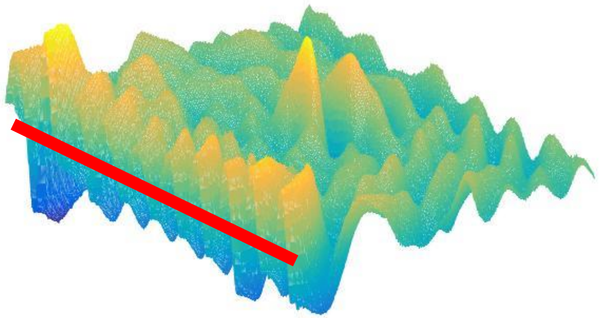
SAFT

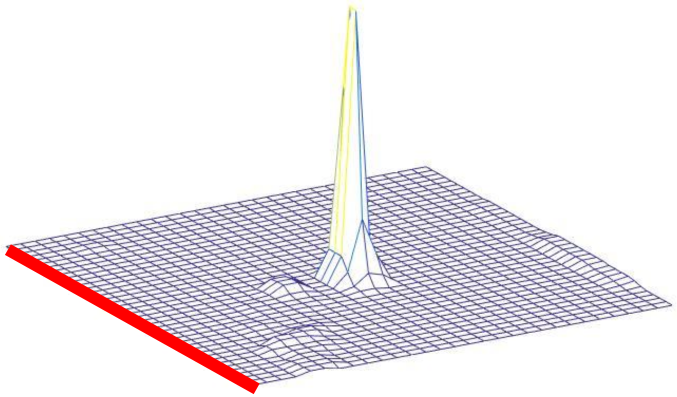


MBIR



10 Transducers equally spaced



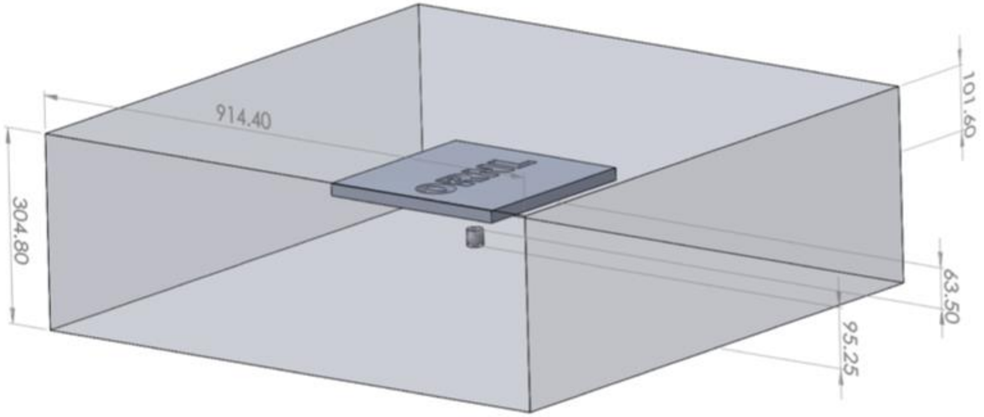


MBIR
- Identifies noise and artifacts
- Easy to interpret
- Clean and clear reconstruction

Empirical Data Reconstruction

Second Experiment:

- Three cement samples
  - Control with no features
  - Flat steel plate
  - Steel plate with ORNL text engraved and steel ball
- Field of view: 40 cm wide, 30 cm deep.
- Acquired initial data set
- The reconstruction of these samples are in progress …



Closing Remarks

Achievements:

- Functional MBIR algorithm for ultrasonic synthetic and empirical data.
- Upgraded forward model, able to compensate for direct arrival signal.
- Better suppression of noise and artifacts compared with SAFT.

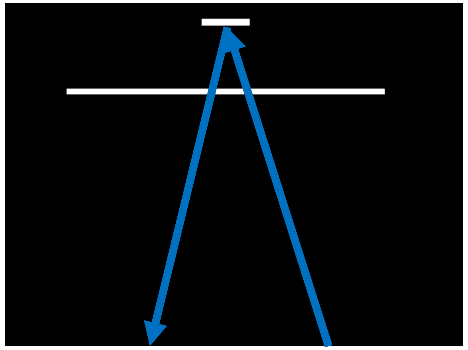
Challenges:

- Dependencies caused by frontal pixels
- Multiple reflection effects

Future work:

- More empirical experiments
- Non-homogeneous anisotropic non-linear forward model

Ground truth



MBIR